

Chapter 10 Appendix

10A Valuing a Lease Renewal Option

10B Simple Model of Optimal Landlord Search for Tenants

10A Valuing a Lease Renewal Option

Leases often contain a variety of options written explicitly into the lease contract, modifying the rights and responsibilities that the landlord and tenant would otherwise face. Some options belong to the tenant, and some to the landlord. In general, it is rather complicated to quantify fully and rigorously the effect such options have on the value of the lease to either party. The most sophisticated approach involves using financial option valuation theory (OVT). Applying OVT to effective rents is out of the scope of this book. (However, we will apply OVT to redevelopments in **Chapter 16**.) A less rigorous, more intuitive technique, decision tree analysis (DTA), is sometimes used to quantify approximately option values in this context. This supplemental material will demonstrate this approach for a simple numerical example of a tenant's renewal option.

Consider again the example of five-year lease A from **section 10.3.2**, with a nominal rent of \$20/SF. Recall that we estimated the effective rent of this lease at \$15.44 from the landlord's perspective, using a discount rate of 7%. Suppose the lease also grants the tenant an option to renew after five years, for another five years, at \$20/SF. The effect of this renewal on the effective rent from the landlord's perspective can be quantified in four computational steps.

Step 1. Describe the subjective probability distribution of market rents when the option matures based on current information. In our example, suppose the landlord believes that at the end of five years (the lease first term expiration date), there is a 50% chance that rents in the relevant space

market will be \$22/SF and a 50% chance that they will be \$18/SF. This expectation is depicted in a binomial branch diagram in **Exhibit 10A.1-a**.¹

EXHIBIT 10A.1-a Decision Tree Approach to Valuing a Lease Renewal Option: Expected Rental Market Conditions and Probabilities for the Example Lease A

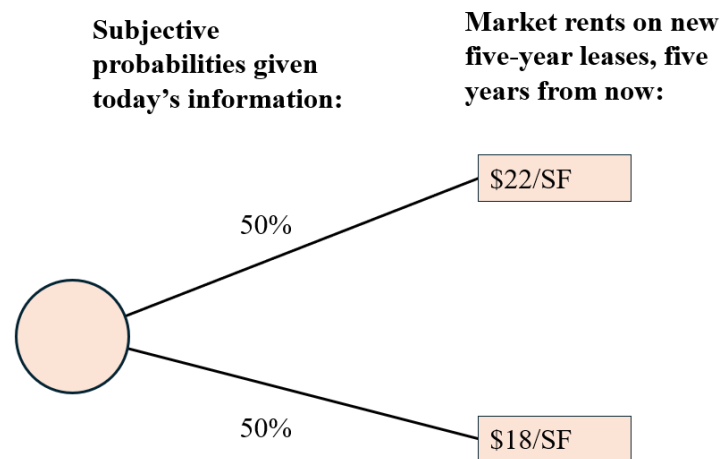
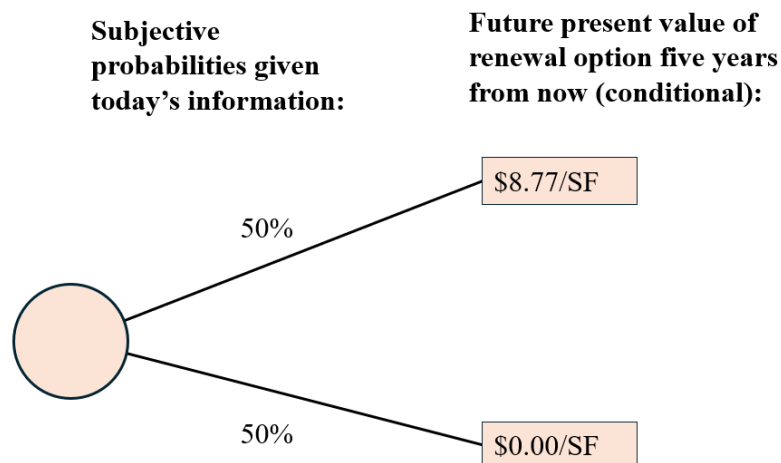


EXHIBIT 10A.1-b Decision Tree Approach to Valuing a Lease Renewal Option: Conditional Future Values and Probabilities in the Example Lease A



¹ In general, the trees in DTA can have more than two branches. Each node represents a decision or valuation point, and the branches going forward in time from each node specify mutually exclusive, exhaustive subsequent states conditional on the node from which they branch. Thus, the probabilities across all branches coming out of each node must sum to 100%, as these are probabilities conditional on the node having been arrived at.

Step 2. Quantify the present value of the renewal option under each future scenario. Assuming the discount rate is still 7% if market rents are \$22/SF, then the option to renew the lease for five years at \$20/SF will be worth:²

$$LPV_5 = (\$22 - \$20) + \frac{\$22 - \$20}{1.07} + \frac{\$22 - \$20}{1.07^2} + \frac{\$22 - \$20}{1.07^3} + \frac{\$22 - \$20}{1.07^4} = \$8.77$$

Note that the landlord loses (and tenant receives) only the difference between the projected market rent (\$22) and the option rent (\$20) if the tenant elects to exercise the renewal option. Also note that as the rent is paid in advance (with no up-front concessions presumed for a new lease), the exercise of the renewal option has its initial \$2 incremental impact at the end of Year 5 (beginning of Year 6) and so is not discounted in the computation of the Year-5 present value.

On the other hand, if market rents are \$18/SF as of the end of Year 5, then the lease renewal option will be worth nothing. There would be no point in the tenant renewing at \$20 when she can get a lease for \$18 in the then-prevailing market. So, the option simply would not be exercised. The decision tree representation of the conditional option value as of the end of Year 5 is shown in **Exhibit 10A.1-b**.

Step 3. Quantify the risk-adjusted present value today (time 0) of the future option value. This is done in two steps that can usually be done in either order.³

Step 3a. Discount each conditional future value back to the present. Normally, a pretty high opportunity cost of capital (OCC) should be used in this discounting because options tend to be pretty risky, especially if they are not deeply “in the money” (i.e., almost certain to be exercised). For example, if existing lease payments are discounted at 7%, and unlevered property cash flows are generally at, say, 9%, then future option values might discount at, say, 15%:⁴

² Some indications of the reasonable expectation of forward short-term interest rates can be obtained from an examination of the yield curve in the bond market only after accounting for the bond yield’s interest rate risk premium component. See the discussion on yield curves in **Part V** as well.

³ The order does not matter because of the distributive property of addition and multiplication: $[(0.5)x + (0.5)y]/(1 + r) = (0.5)x/(1 + r) + (0.5)y/(1 + r)$.

⁴ Note that the risk in the option will probably change over time as new information arrives relevant to the likelihood of option exercise. The difficulty of knowing the correct discount rate to employ in this step is a major weakness in

$$PV(\$8.77 \text{ in } 5 \text{ yrs.}) = \$8.77/(1.15)^5 = \$4.36$$

$$PV(\$0.0 \text{ in } 5 \text{ yrs.}) = \$0/(1.15)^5 = \$0$$

Step 3b. Sum across the possible scenario present values, weighted by their present subjective probabilities of occurrence. In this case,

$$(0.50)\$4.36 + (0.50)\$0 = \$2.18$$

This gives the present value of the lease renewal option today. The landlord, who gives the option to the tenant, receives a negative value (or positive cost).

Step 4. Convert the renewal option present value to an impact on effective rent. Using the annuity formula (for cash flows in advance), a PV of \$2.18 equates to a five-year 7% annuity of \$0.50 per year:

$$\$0.50 = (7\%)(\$2.18)/\{(1 + 7\%)[1 - 1/(1 + 7\%)^5]\}$$

Thus, the renewal option will reduce the landlord's effective rent of lease A from \$15.44 to \$14.94..⁵

the DTA approach (as compared, for example, to the more rigorous, but much more technical and complex, OVT-based approach).

⁵ If the option alternatives involve different horizons, then the conversion to annualized values should be done in Step 2 rather than Step 4.

10B Simple Model of Optimal Landlord Search for Tenants

This appendix presents a simple numerical example of optimal landlord behavior when searching for tenants in the form of a model of optimal rent asking. The analysis here may also be of interest as an example of how modern business quantitative analysis techniques can be used to shed light on basic commercial property management and investment decision issues. More generally, a model like this may be applied beyond the leasing decision to shed some light, for example, on optimal behavior in the purchase or sale of investment property. However, it is important to note that the model presented here is not an equilibrium model. It considers only one side of what is more realistically a two-sided search market in most situations.

Consider the following simplified model of optimal asking rent for a landlord with an empty space. In particular, suppose the following conditions characterize the search problem:

1. Potential tenants “arrive” (or are found) randomly at an average rate of one per month. The expected wait time until the first potential tenant is found is one month, until the second is found in two months, etc..⁶
2. The ex-ante probability distribution of the maximum rent each potential tenant will accept (the tenant’s reservation rent) is a normal probability distribution with a mean of \$10/SF/year and a standard deviation of $\pm \$1/\text{SF}/\text{year}$. All leases are for five-year lease terms, annual rent, and payments at the beginning of the year. The landlord only finds out what each tenant is willing to pay when that tenant “arrives.”
3. If the tenant refuses the landlord’s asking rent, the landlord must wait until the next potential tenant arrives, and the space remains vacant during the wait time. When the space leases, it will always lease at the landlord’s asking rent..⁷
4. This process repeats (no renewals) when a lease expires in perpetuity.

⁶ This is what is known as a Poisson arrival process. It is characterized by being “memoryless,” that is, one arrival is not influenced by another arrival. Obviously, this simplifies the real-world process by which landlords and tenants search for each other, but it captures much of reality’s essence. Mathematically, the probability that the first potential tenant will be found t periods from now is $\lambda e^{-\lambda t}$, where e is the base of natural logs (approximately 2.7183), and λ is the average rate of arrival per period. In a Poisson process, the expected time until the first arrival is the inverse of the arrival rate, $1/\lambda$, and the expected time until the n th arrival is n/λ . With time measured in years, condition (1) postulates $\lambda = 12$ arrivals /year.

⁷ Conditions (2) and (3) together represent the lease negotiation process. Once again, the model abstracts and simplifies from the real world while still capturing much of the essence.

5. The intralease discount rate is 8%; the interlease discount rate is 12%. (See **Chapter 4** for a discussion on the difference between intralease and interlease risk.)

Given this situation, we want to answer the question of what asking rent will maximize the present value of the building. To answer this question, we can specify the following analytical model. Let:

- A = Asking rent
- $N(A;10,1)$ = Cumulative normal probability less than A when the mean is ten and STD is 1.⁸
- p = Probability tenant refuses landlord's offer = $N(A;10,1)$
- w = Expected wait time (in years) until space is leased (average length of vacancy period between leases)
- L = PV of each lease at the time of signing (intralease discounting)
- V = PV of the building (perpetuity)
- vac = Expected vacancy rate for building

Now consider the expected wait time, w . As p is the probability that any one potential tenant will refuse the landlord's offer, $(1 - p)$ is the probability that any one tenant will accept the offer. The expected time until the first tenant arrives is $(1/12)$ years. Similarly, $(n/12)$ is the expected time (in years) until the n^{th} potential tenant arrives. However, if the first one accepts the landlord's offer, the second potential tenant will never arrive. And the third tenant will never arrive if the second one accepts the landlord's offer. Consider the expression:

$$(n/12)(p^{n-1})(1 - p)$$

This expression is the product of three factors: $(n/12)$ is the expected time until the n^{th} potential tenant arrives; p^{n-1} is the probability that all $n - 1$ previous arrivals have refused the landlord's offer (i.e., the probability that the landlord has to wait for the n^{th} arrival); and $(1 - p)$ is then the probability that the n^{th} arrival will accept the landlord's offer. The unconditional expected waiting time for the landlord, w , is thus a sum of expressions like $(C1)$, summed across all possible number

⁸ For example, in Excel, the function `NORMDIST(A,10,1,1)` gives this quantity. We used a similar function when performing our simulation analysis in **Chapter 9**.

of potential tenant arrivals, which is, in principle, infinite ($n = 1, 2, \dots, +\infty$). Therefore, the landlord's expected waiting time until she finds a tenant for a given space is

$$w = \frac{1}{12}(1 - p) + \frac{1}{12}p(1 - p) + \frac{3}{12}p^2(1 - p) + \dots$$

This equation is an infinite series; however, it has a simple finite value, namely,⁹

$$w = \frac{1}{(1 - p)^{12}}$$

Lease value as a function of the asking rent A is given by the annuity formula (with payments in advance);

$$L = \frac{A(1.08)[1 - (\frac{1}{1.08})^5]}{0.08}$$

The property value, V , is then given by the formula for the sum of an infinite geometric series as the perpetuity of leases, each expected to occur $5 + w$ years after the previous lease was signed, discounting at the interlease discount rate between leases. The first lease is expected to be signed w years from time zero:

$$V = \frac{L/(1.12)^w}{1 - (1/1.12)^{w+5}}$$

The average vacancy in the space is simply given by the expected time between leases divided by the lease term (five years):

$$vac = \frac{w}{w + 5}$$

This model can be easily solved quantitatively in a computer spreadsheet. Try some values of the asking rent, A , until you find the one that maximizes the present value of the space, V . Here is the result of such an iterative search procedure, starting at the average potential tenant reservation rent of \$10/SF, and working up from there.

⁹ There is some intuition in this result. A Poisson arrival process with arrival rate λ in which each arrival has an independent probability $(1 - p)$ of being a "hit" is equivalent to a Poisson arrival process of hits with arrival rate $\lambda(1 - p)$. In a Poisson process, the expected time until the first arrival is the inverse of the arrival rate. Thus, the expected time until the arrival of the first hit is $1/[\lambda(1 - p)]$, or in the present case, $1/[12(1 - p)]$.

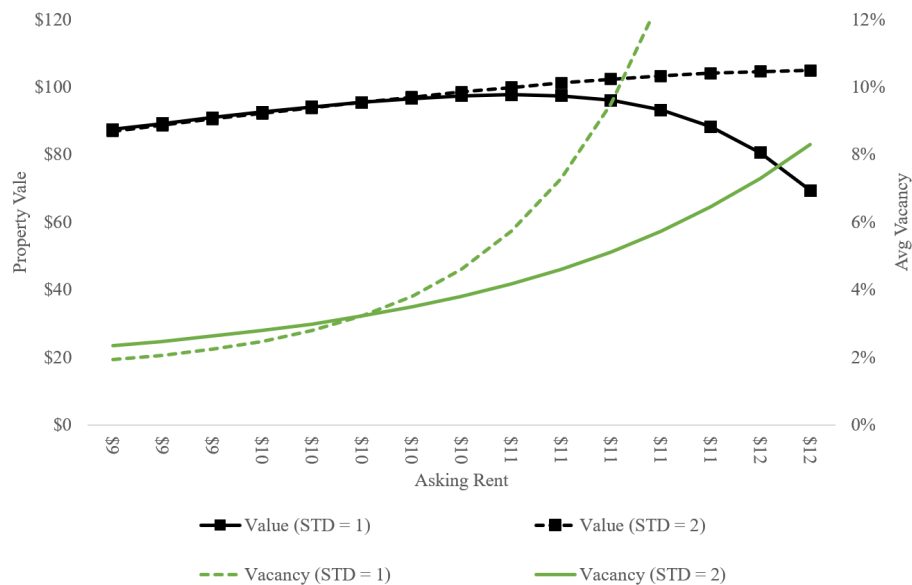
A	V
\$10.00	\$95.48
\$11.00	\$96.04
\$12.00	\$54.64
\$10.90	\$96.87
\$10.80	\$97.40
\$10.70	\$97.68
\$10.60	\$97.75
\$10.50	\$97.65

Thus, in this case, the optimal asking rent is \$10.60/SF (to the nearest dime), which gives a building value of \$97.75/SF. At this asking rent the expected waiting time to find a tenant that takes the rent (expected vacant period between leases) is $w = 0.304$ years, that is, between three and four months. This implies an optimal long-term average vacancy rate of $\text{vac} = .304/5.304 = 5.7\%$.¹⁰

Now repeat this example only suppose the landlord's ex-ante uncertainty surrounding the rent the potential tenants will take is doubled. That is, assume everything is the same except the standard deviation of the normal probability distribution is \$2/SF instead of \$1/SF. Thus, $p = N(A;10,2)$. We now find that the optimal rent is \$11.80/SF, giving a building value of \$104.87/SF with an average vacant period of 0.453 years (about five months) and an average vacancy rate of 8.3%.

EXHIBIT 10B.1 Landlord Optimal Asking Rent and Vacancy Rate as a Function of Rental Market Uncertainty (Assumptions: Tenant maximum rent distribution normal with mean = \$10/SF, lease term = five years, intralease discount rate = 8%, interlease discount rate = 12%.)

¹⁰ At the \$10.60 asking rent, the probability of potential tenant refusal is $p = N(10.60;10,1) = 72.6\%$, which gives an acceptance probability of $1 - 0.726 = 0.274$. This gives an expected wait between leases of $w = 1/[(0.274)12] = 1/3.288 = 0.304$ years.



Note that the optimal asking rent, the optimal average vacancy rate, and the building value all increase with the uncertainty or range in the maximum rent the potential tenants are willing to accept. This is a general result. The two cases examined here are shown in the graph in Exhibit C30-1. (The numbers on the left-hand vertical scale refer to \$/SF for the top [solid] lines indicating property value, and this same scale refers to the average percent vacancy for the bottom [dashed] lines. The triangle markers indicate the case with the lower rent uncertainty.) The general shapes of the curves in this graph also provide a general result for typical realistic numbers. From the previous analysis, we can derive several results about the optimal asking rent for the landlord in this simple model:

- Other things being equal, the optimal asking rent is higher the more uncertainty there is about the rental market.
- The greater the rental market uncertainty, the more “forgiving” is the negative impact on property value due to an equal dollar magnitude error on the landlord’s part in not selecting the optimal asking rent (i.e., the curve is “flatter” for higher variance in the rent distribution, at least on the upside).
- The effect on property value is relatively forgiving for erring on the side of asking for a rent that is too low. At the same time, the negative value impact of asking for a rent too

high can be much more severe, especially when there is little uncertainty about the rental market.

- Other things being equal (such as the mean expected tenant reservation rent) and assuming optimal landlord behavior, the property is more valuable the greater the uncertainty in the rental market, but this value effect is small even though our model ignores the effect of rent uncertainty on the interlease discount rate (which might dampen or reverse this result).

Although the model on which these conclusions are based simplifies reality, the first three conclusions are fairly robust if one interprets them broadly or “figuratively.” For example, they can be paraphrased in the following two more general principles of optimal tenant search and leasing strategy for a landlord:

1. Be a bit daring and aggressive in pursuing good leasing deals if you are uncertain about the rental market. (This is a generalization of the first two preceding points.)
2. Be conservative and play it safe if the landlord is very risk-averse or if the rental market is very obvious, with little uncertainty about market rents. (This is a generalization of the third previous point.)